On the determination of surface flow paths from gridded elevation data

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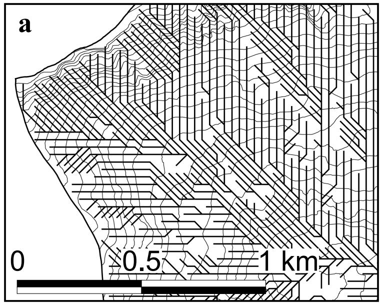
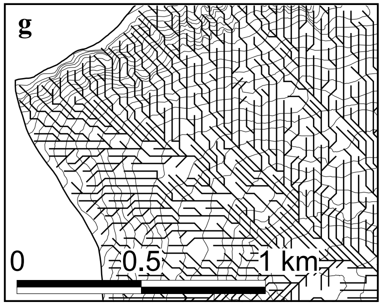
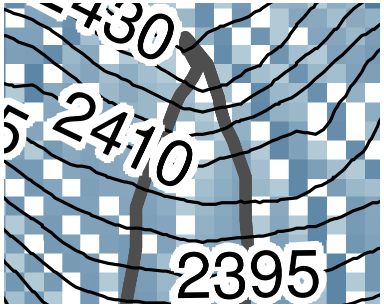
Background and Motivation for This Study



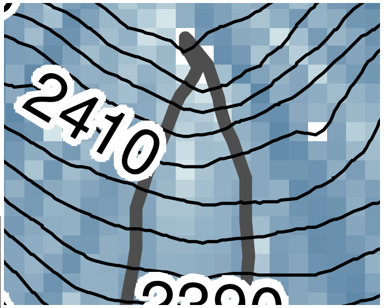
As noted by Moore and Grayson (1991, WRR), the D8 method has two major limitations: (1) the flow direction from each grid cell is restricted to only eight possibilities, and (2) a drainage area which originates over a two-dimensional cell is treated as a point source (nondimensional) and is projected downslope by a line (one-dimensional).



Path-based methods does not affect limitation (1), but solves, at least in part, the problem of aligned flow paths through an improved selection of single flow directions (Orlandini et al., 2003, WRR).



The effects of limitation (2) on computed drainage areas remain.

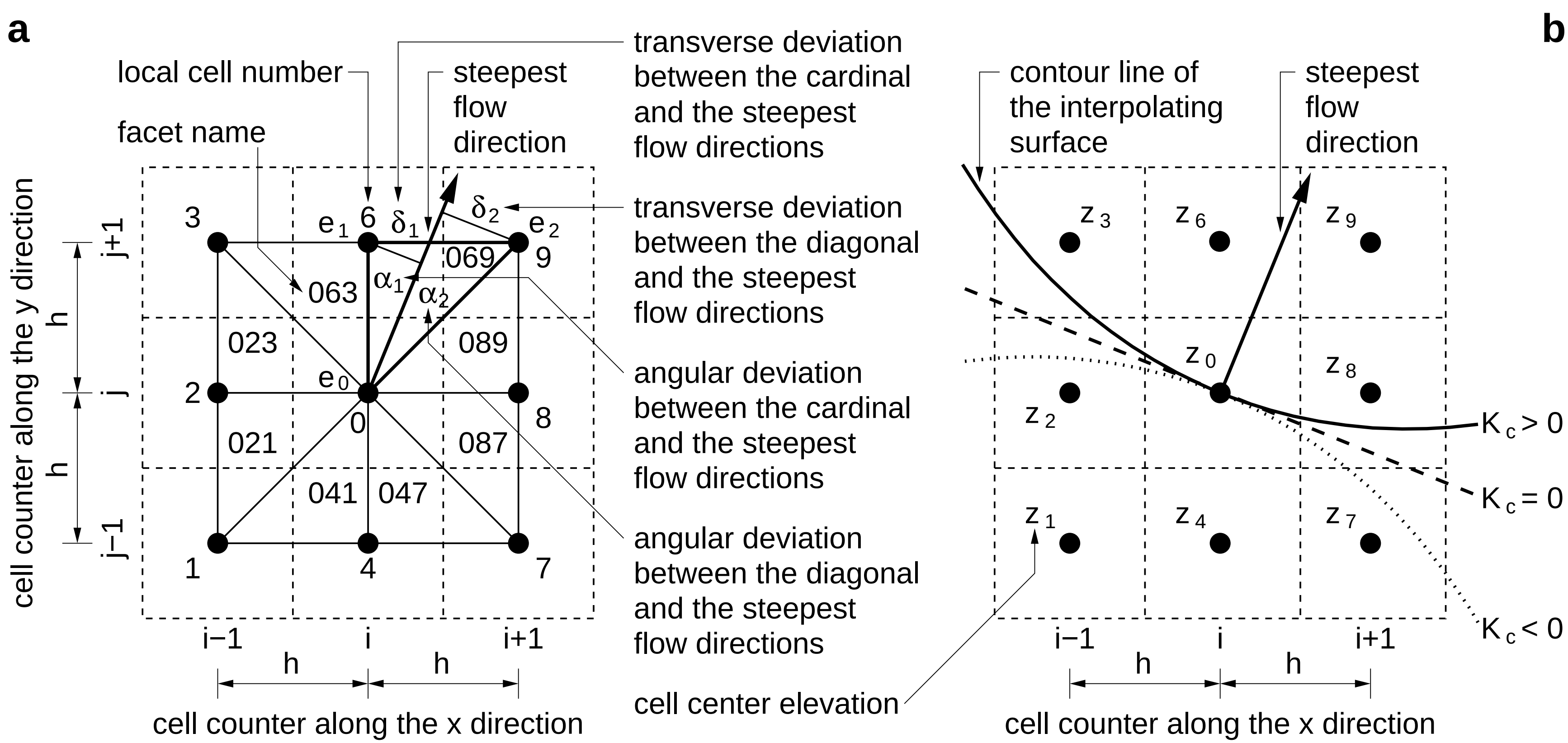


Multiple flow direction methods were introduced to mitigate the effects of limitations (1) and (2), but they are not consistent with the concept of drainage basin (Quinn et al., 1991, HP; Tarboton, 1997, WRR; Seibert and McGlynn, 2007, WRR). This inconsistency has not been adequately investigated so far in an accurate manner.

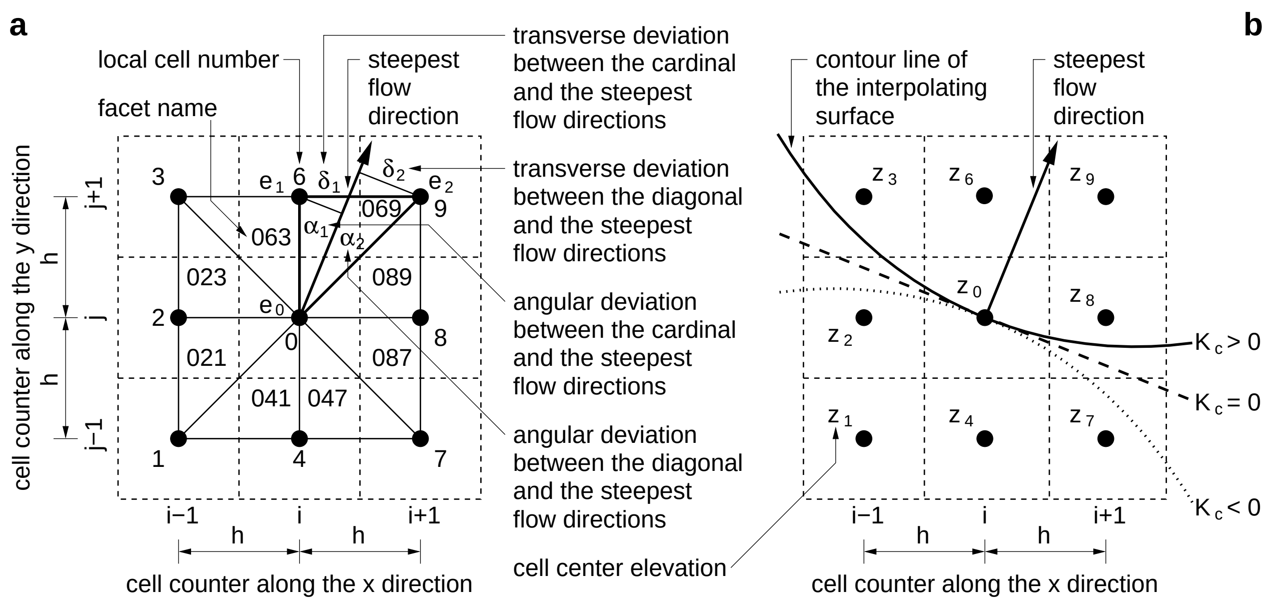
Mathematical Formulation

The mathematical formulation developed in this study is based on the use of:

- Triangular facets introduced by Tarboton (1997, WRR)
- Path-based analysis introduced by Orlandini et al. (2003, WRR)
- Plan curvature computation introduced by Zevenbergen and Thorne (1987, ESPL)



Mathematical Formulation



	Facet							
	021	023	063	069	089	087	047	041
e_0	$e_{i,j}$	$e_{i,j}$	$e_{i,j}$	$e_{i,j}$	$e_{i,j}$	$e_{i,j}$	$e_{i,j}$	$e_{i,j}$
e_1	$e_{i-1,j}$	$e_{i-1,j}$	$e_{i,j+1}$	$e_{i,j+1}$	$e_{i+1,j}$	$e_{i+1,j}$	$e_{i,j-1}$	$e_{i,j-1}$
e_2	$e_{i-1,j-1}$	$e_{i-1,j+1}$	$e_{i-1,j+1}$	$e_{i+1,j+1}$	$e_{i+1,j+1}$	$e_{i+1,j-1}$	$e_{i+1,j-1}$	$e_{i-1,j-1}$
p_1	2	2	6	6	8	8	4	4
p_2	1	3	3	9	9	7	7	1
σ	$+1$	-1	$+1$	-1	$+1$	-1	$+1$	-1

Mathematical Formulation

Local or Path-based Analyses

$$k = 1$$

$$\delta_1^+(1) = \sigma\, \delta_1(1)$$

$$\delta_2^+(1) = -\sigma\, \delta_2(1)$$

If $|\delta_1^+(1)| \le |\delta_2^+(1)|$, then $p = p_1,\ \delta^+(1) = \delta_1^+(1)$

If $|\delta_1^+(1)| > |\delta_2^+(1)|$, then $p = p_2,\ \delta^+(1) = \delta_2^+(1)$

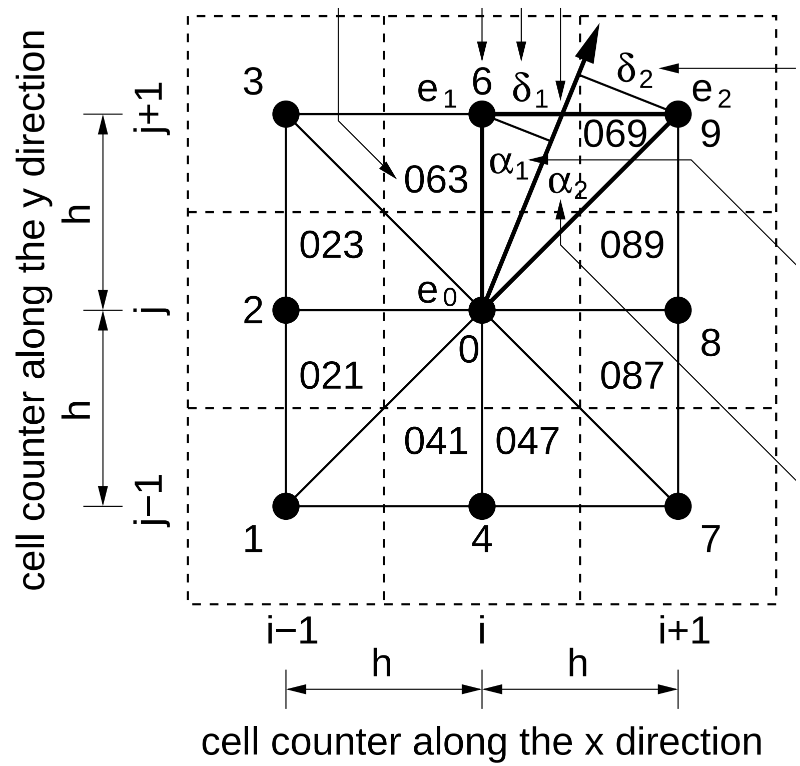
$$k = 2, 3, \ldots$$

$$\delta_1^+(k) = \sigma\, \delta_1(k) + \lambda\, \delta^+(k-1)$$

$$\delta_2^+(k) = -\sigma\, \delta_2(k) + \lambda\, \delta^+(k-1)$$

If $|\delta_1^+(k)| \le |\delta_2^+(k)|$, then $p = p_1,\ \delta^+(k) = \delta_1^+(k)$

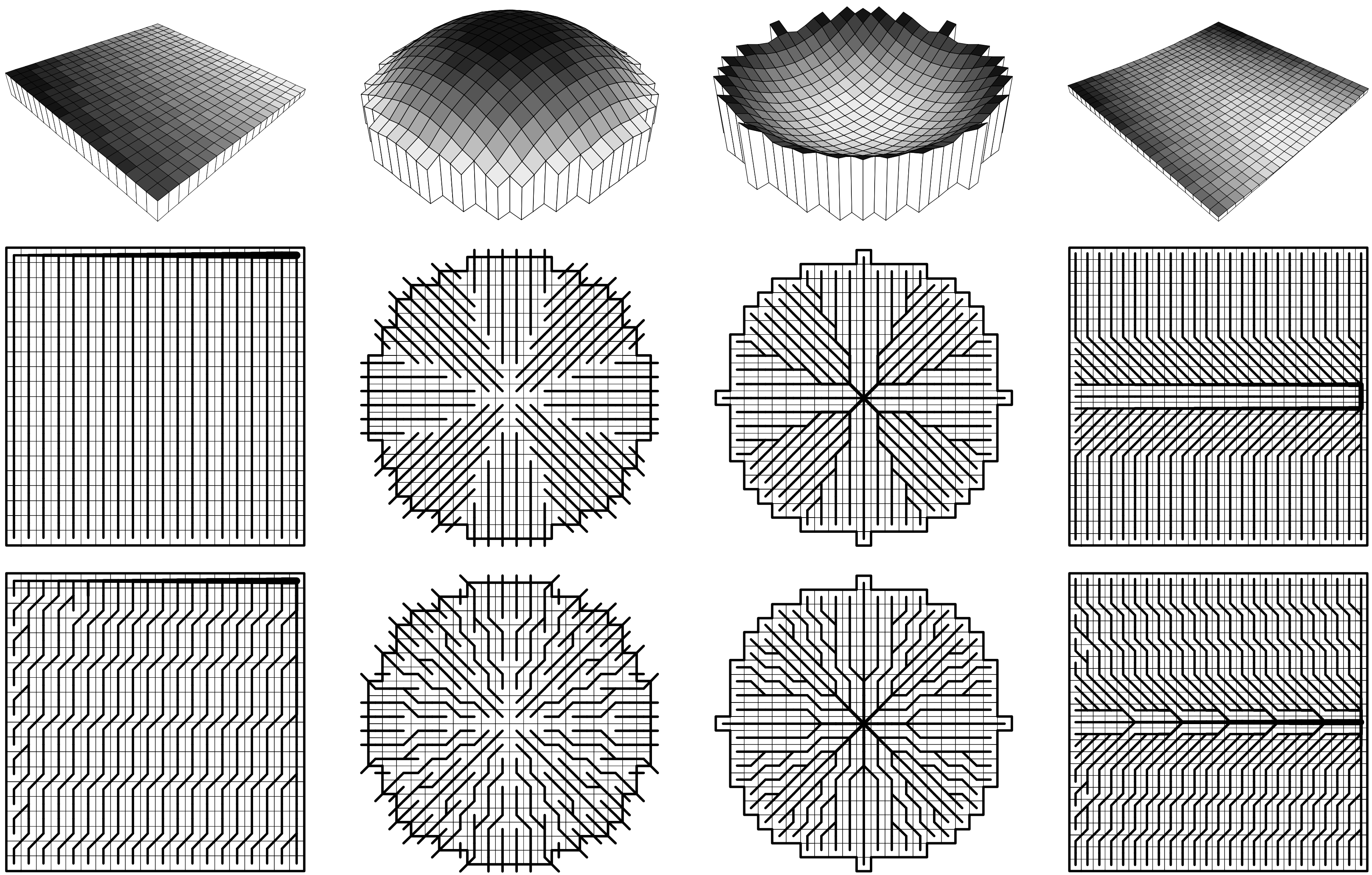
If $|\delta_1^+(k)| > |\delta_2^+(k)|$, then $p = p_2,\ \delta^+(k) = \delta_2^+(k)$



Mathematical Formulation

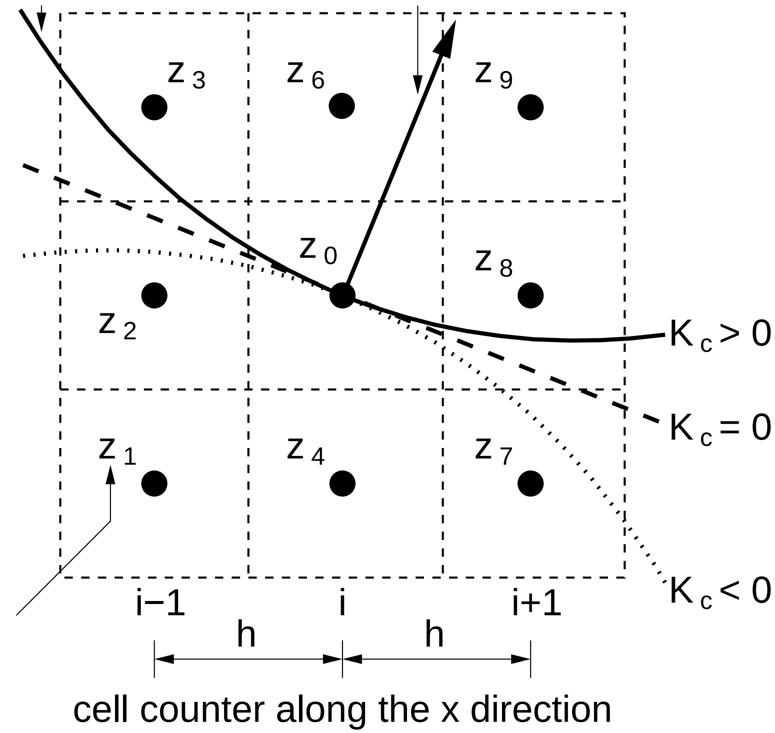
Orlandini et al. (2003, WRR)

Planar Slope | Spheric Mountain | Spheric Crater | Parabolic Valley



Mathematical Formulation

Nondispersive or Dispersive Analyses



$$w_1 = \frac{|\delta_2^+(k)|}{|\delta_1^+(k)| + |\delta_2^+(k)|}$$

$$w_2 = \frac{|\delta_1^+(k)|}{|\delta_1^+(k)| + |\delta_2^+(k)|}$$

$$\delta^+(k-1) = \sum_i w_i\, A_i\, \delta_i^+(k-1) \Big/ \sum_i w_i\, A_i$$

$$K_c = \frac{z_{xx}\, z_y^2 - 2\, z_{xy}\, z_x\, z_y + z_{yy}\, z_x^2}{(z_x^2 + z_y^2)^{3/2}}$$

where $z_x = \partial z/\partial x \approx (z_8 - z_2)/(2\,h)$, $z_y = \partial z/\partial y \approx (z_6 - z_4)/(2\,h)$,
$z_{xx} = \partial^2 z/\partial x^2 \approx (z_8 - 2\,z_0 + z_2)/h^2$, $z_{yy} = \partial^2 z/\partial y^2 \approx (z_6 - 2\,z_0 + z_4)/h^2$, and
$z_{xy} = \partial^2 z/(\partial x \partial y) \approx (-z_3 + z_9 + z_1 - z_7)/(4\,h^2)$.

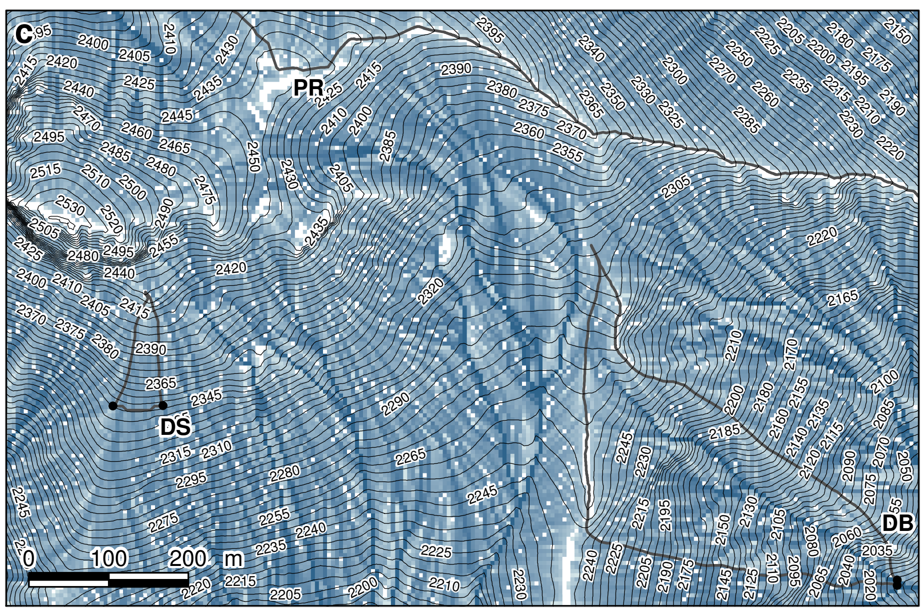
A threshold value for K_c is set equal to K_{ct} and a single or multiple flow direction is used depending on whether $K_c > K_{ct}$ or $K_c \le K_{ct}$, respectively.

Unifying Flow Direction Algorithm

Method	LAD/LTD†, λ	K_{ct}	ND/D/H‡
D8	LAD, $\lambda = 0$	$(-\infty, \min(K_c))$	ND
D∞	LAD, $\lambda = 0$	$[\max(K_c), +\infty)$	D
D8-LTD	LTD, $\lambda = 1$	$(-\infty, \min(K_c))$	ND
D∞-LTD	LTD, $\lambda = 1$	$[\max(K_c), +\infty)$	D
D8/∞-LTD	LTD, $\lambda = 1$	$(-\infty, +\infty)$	H

†LAD: least angular deviation; LTD: least transverse deviation.

‡ND: nondispersive; D: dispersive; H: hybrid.

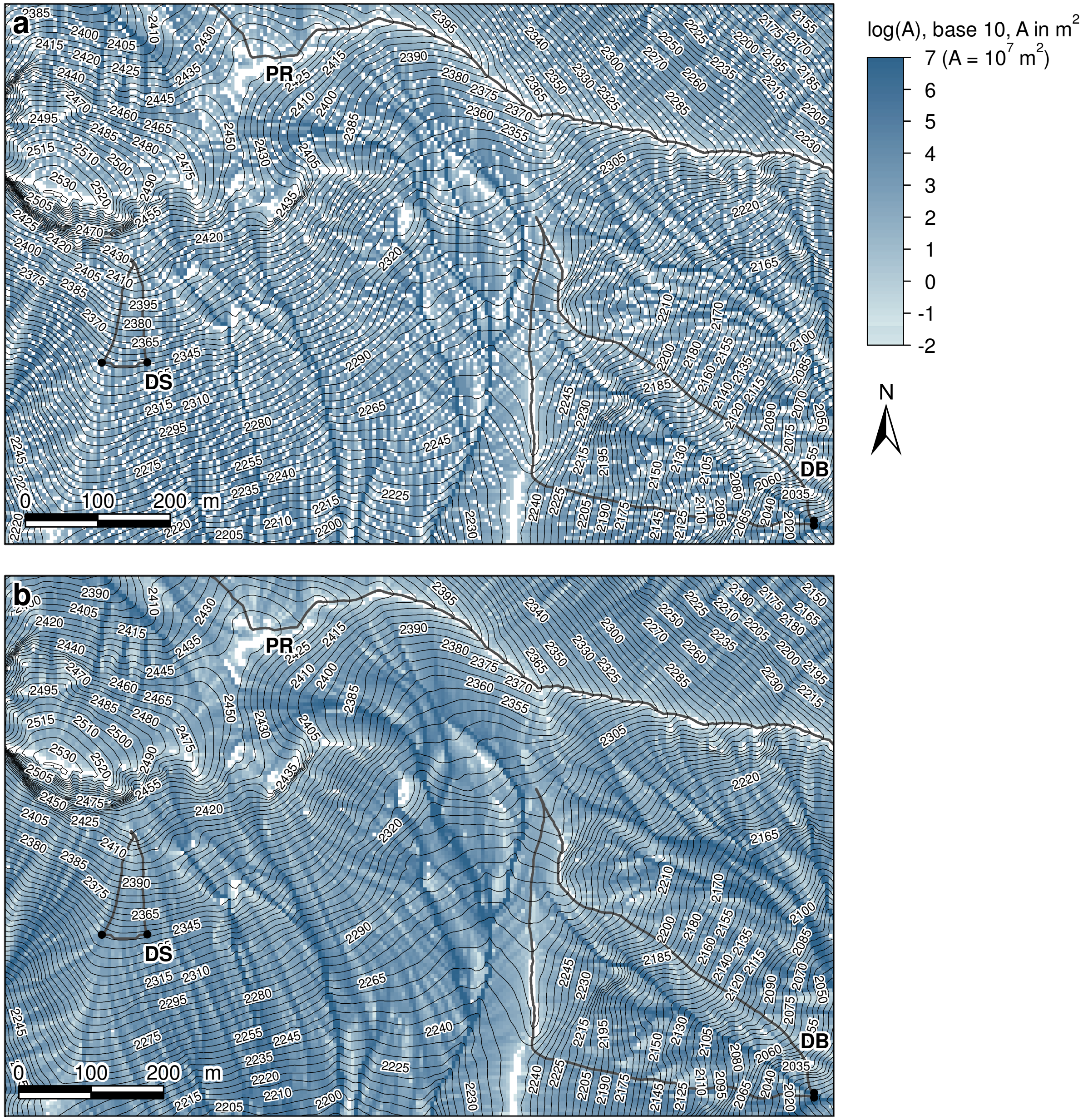


LiDAR Data

Survey carried out by Terrapoint (USA) in the Col Rodella area (Italian Alps):
ALTM system of Terrapoint's proprietary design mounted on a fixed-wing aircraft.
The flight altitude: about 1000 m agl.
Average data density: about 0.5 points per square meter.
Horizontal absolute accuracy: 1 m.
Vertical absolute accuracy: 0.3 m.
Resolution of the gridded elevation data provided: 1 m.

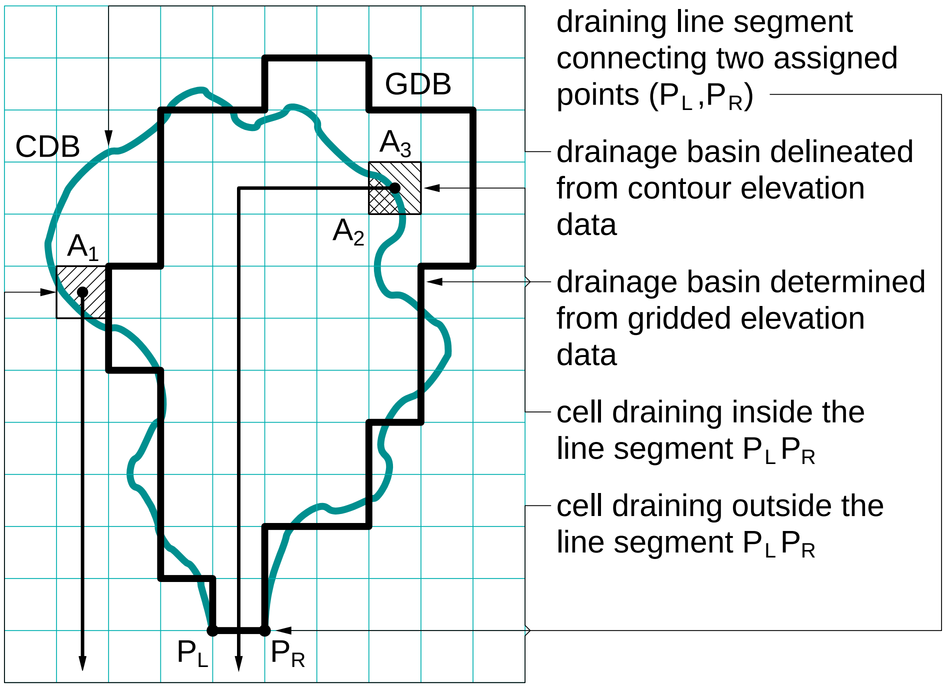
Gridded elevation data were resampled to 5, 10, 50, 100, and 200-m. Contour elevation data with contour intervals of 1, 5, 10, 20, and 50 m were also generated.

Validation Using Drainage Areas



Although drainage areas have widely been used to evaluate flow direction algorithms, correct drainage areas do not necessarily imply correct surface flow paths across the upslope terrain. A meaningful application of drainage areas is made in this study by using drainage basins and slopes delineated from contour elevation data as a reference, and by evaluating relative errors accounting for both magnitude and location of the drainage areas accumulated along surface flow paths obtained from gridded elevation data.

Validation Using Contour Elevation Data



1. A region containing both the CDB and the GDB is considered. A weight equal to the degree of belonging to the CDB is assigned to each grid cell. Surface flow paths are determined from gridded elevation data and the area drained by the line segment $P_L P_R$ is computed. The result provides area A_2.
2. Area A_1 is computed as the difference between the area of the CDB $(A_1 + A_2)$ obtained from contour elevation data and area A_2 obtained from the previous step.
3. To each grid cell a weight equal to 1 is assigned. Surface flow paths are determined from gridded elevation data and the area drained by the line segment $P_L P_R$ is computed. The result provides area $(A_2 + A_3)$.
4. Area A_3 is computed as the difference between area $(A_2 + A_3)$ obtained from the previous step and area A_2 obtained from the first step.

Type 1 Relative Error

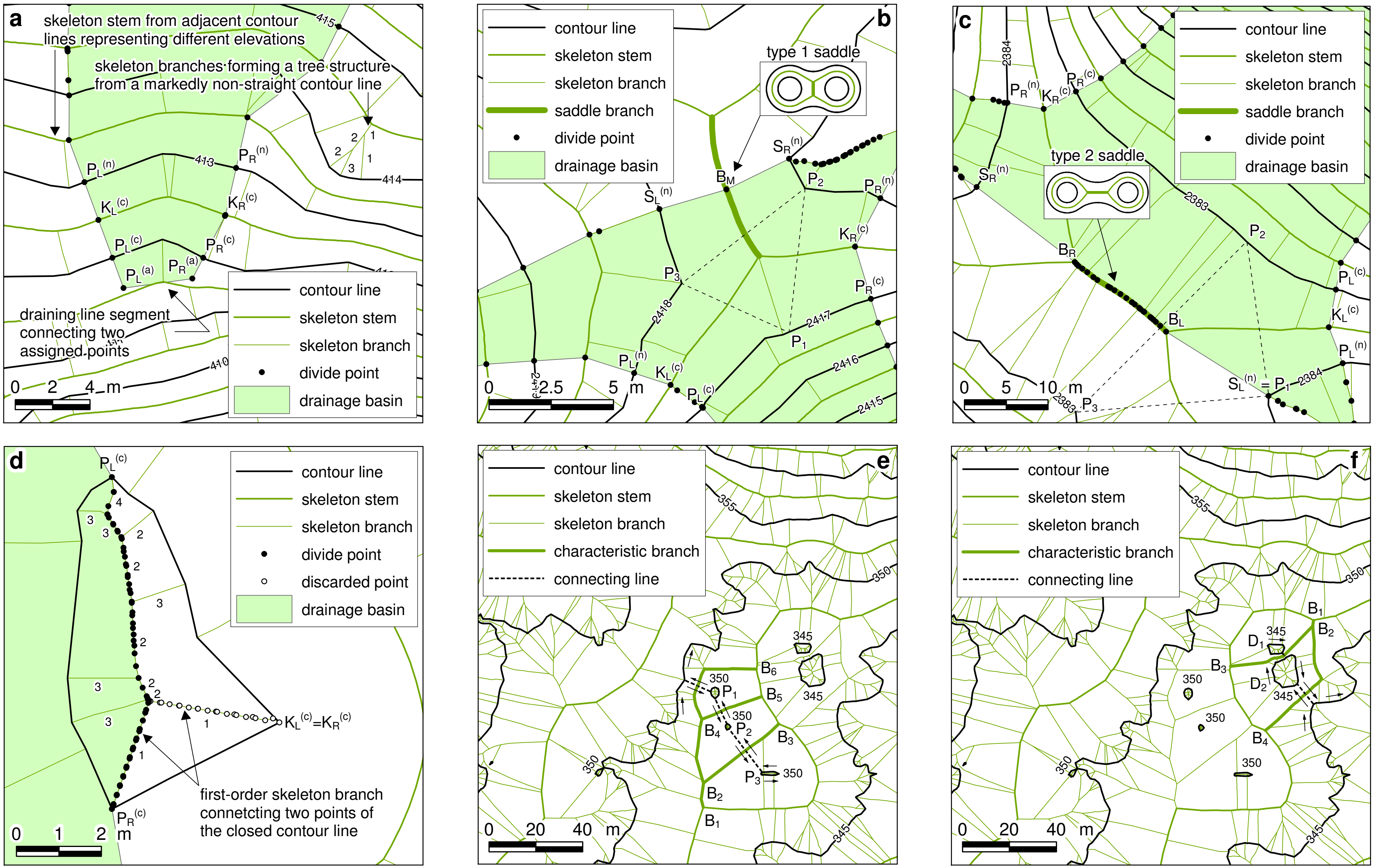
$$E_1 = \frac{|A_1 - A_3|}{A_1 + A_2}$$

Type 2 Relative Error

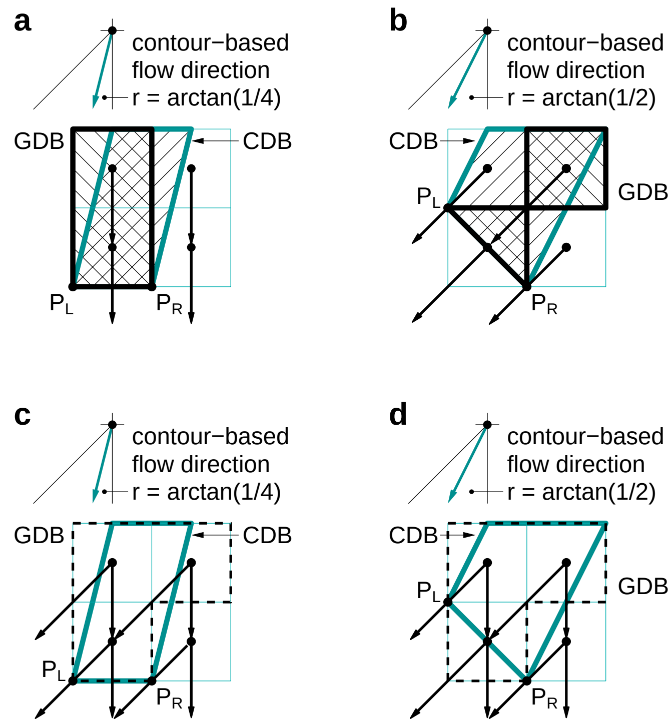
$$E_2 = \frac{A_1 + A_3}{A_1 + A_2}$$

Validation Using Contour Elevation Data

Moretti and Orlandini (2008, WRR)

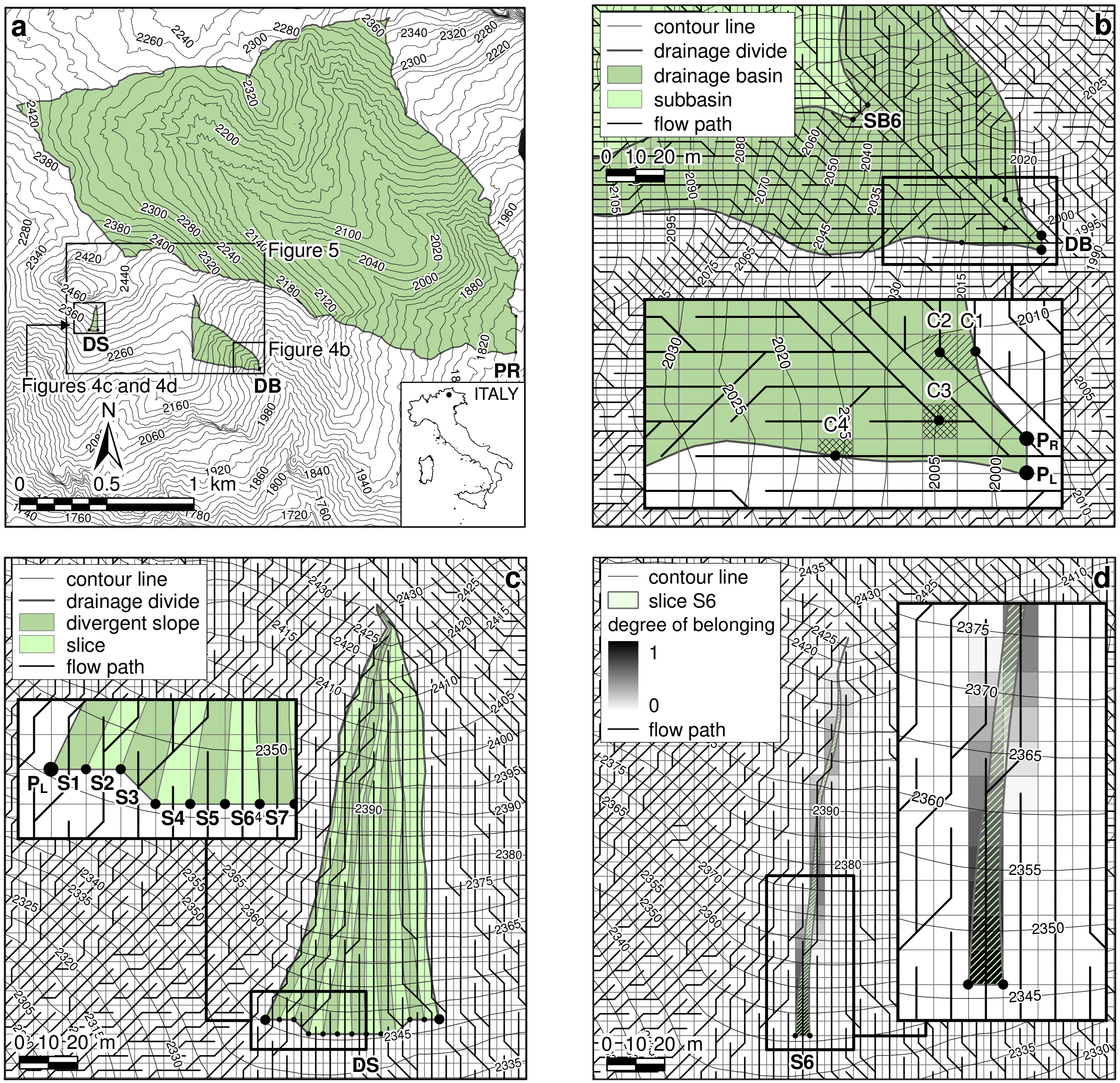


Single and Multiple Flow Directions



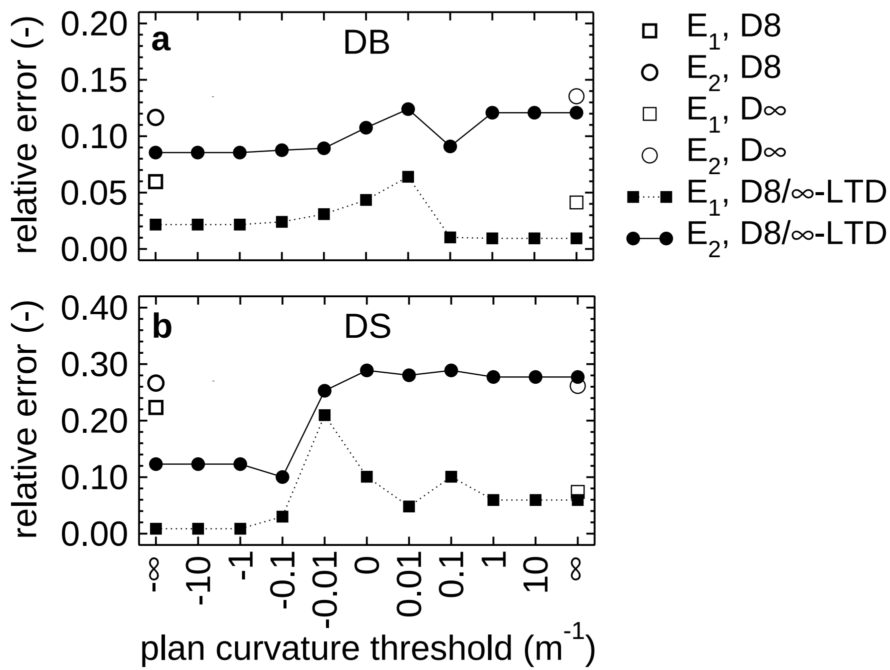
Case	r, rad	α_1, rad	α_2, rad	w_1, -	w_2, -	A_1, h^2	A_2, h^2	A_3, h^2	E_1, -	E_2, -
a	0.245	0.000	0.785	1.000	0.000	0.500	1.500	0.500	0.000	0.500
b	0.464	0.785	0.000	0.000	1.000	1.000	1.250	0.250	0.333	0.556
c	0.245	0.245	0.540	0.688	0.312	1.022	0.978	0.398	0.312	0.710
d	0.464	0.464	0.322	0.410	0.590	1.000	1.250	0.250	0.333	0.556

Real Study Cases

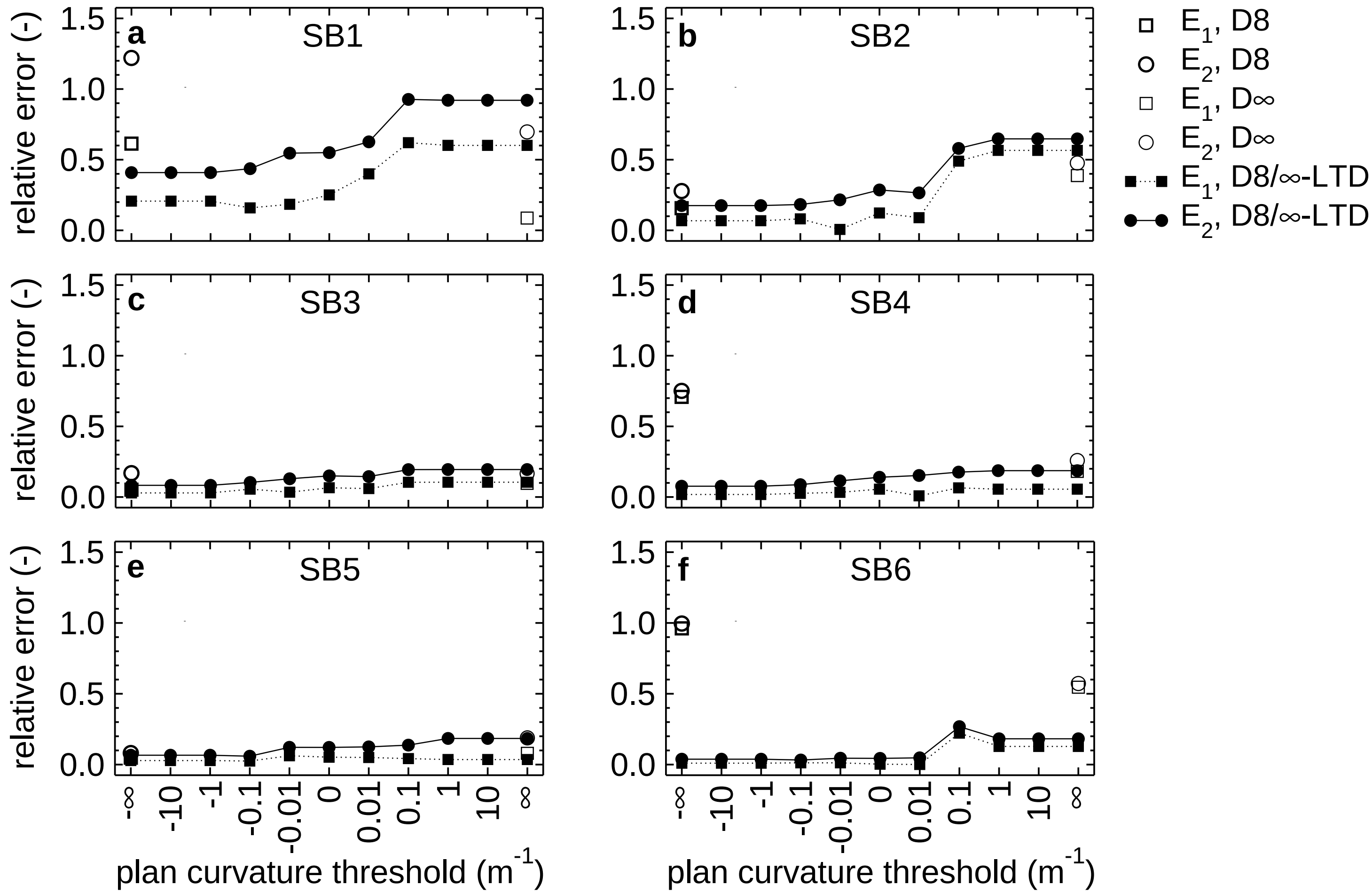


Drainage systems having different size and morphological features are selected within the Col Rodella area as shown in Figure a. A small convergent drainage basin (DB) and a divergent slope (DS) are considered. In addition, six subbasins (SB1–SB6) are selected within the DB (SB6 shown in Figure b). Then, the DS is subdivided into twelve slices (S1–S12, Figures c and d), each drained by a single grid cell. Finally, a relatively large drainage basin denoted as Pra Rodella (PR in Figure a) is finally considered, with grid cell size of 5, 10, 50, 100, and 200 m.

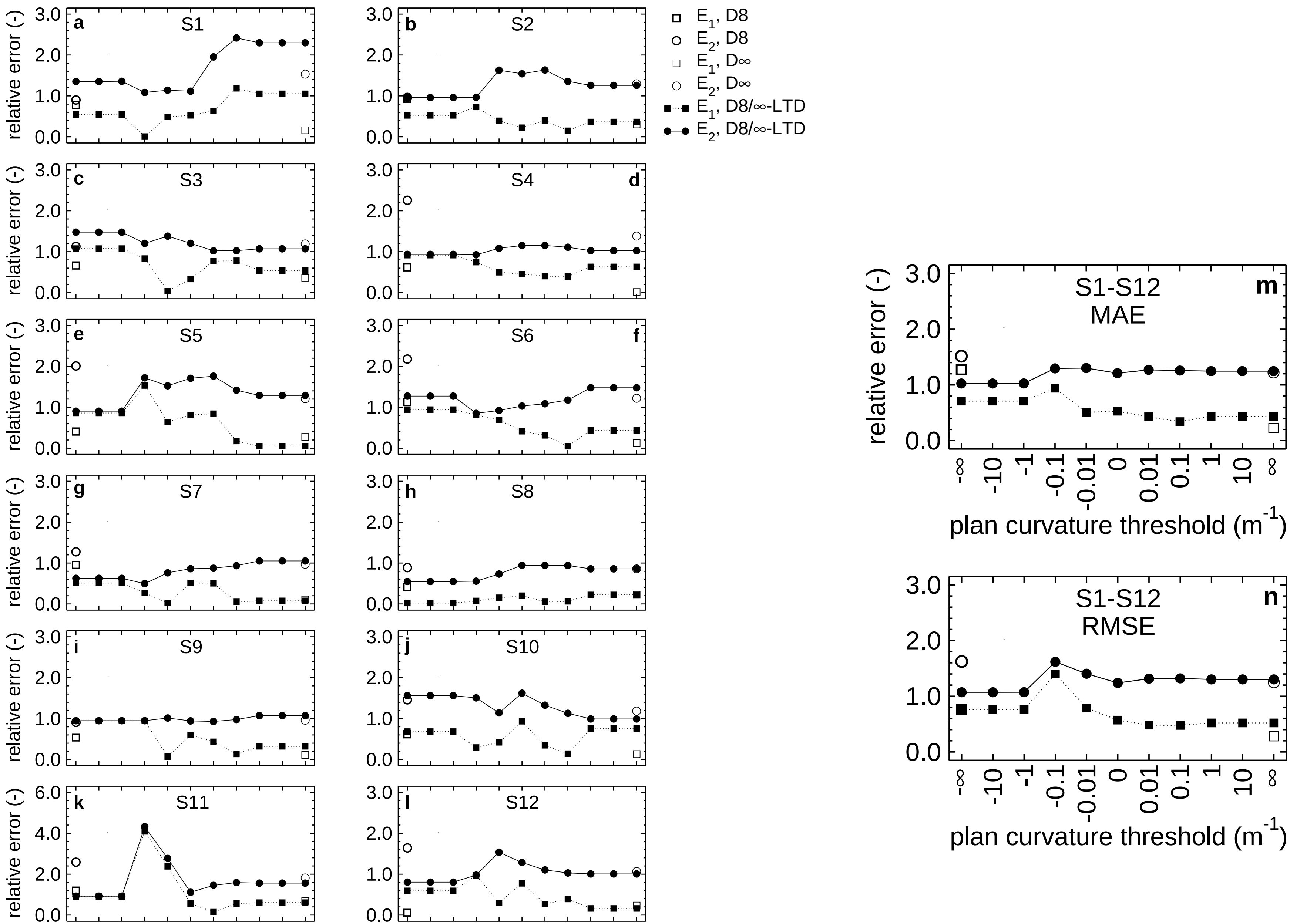
Drainage Basin and Drainage Slope



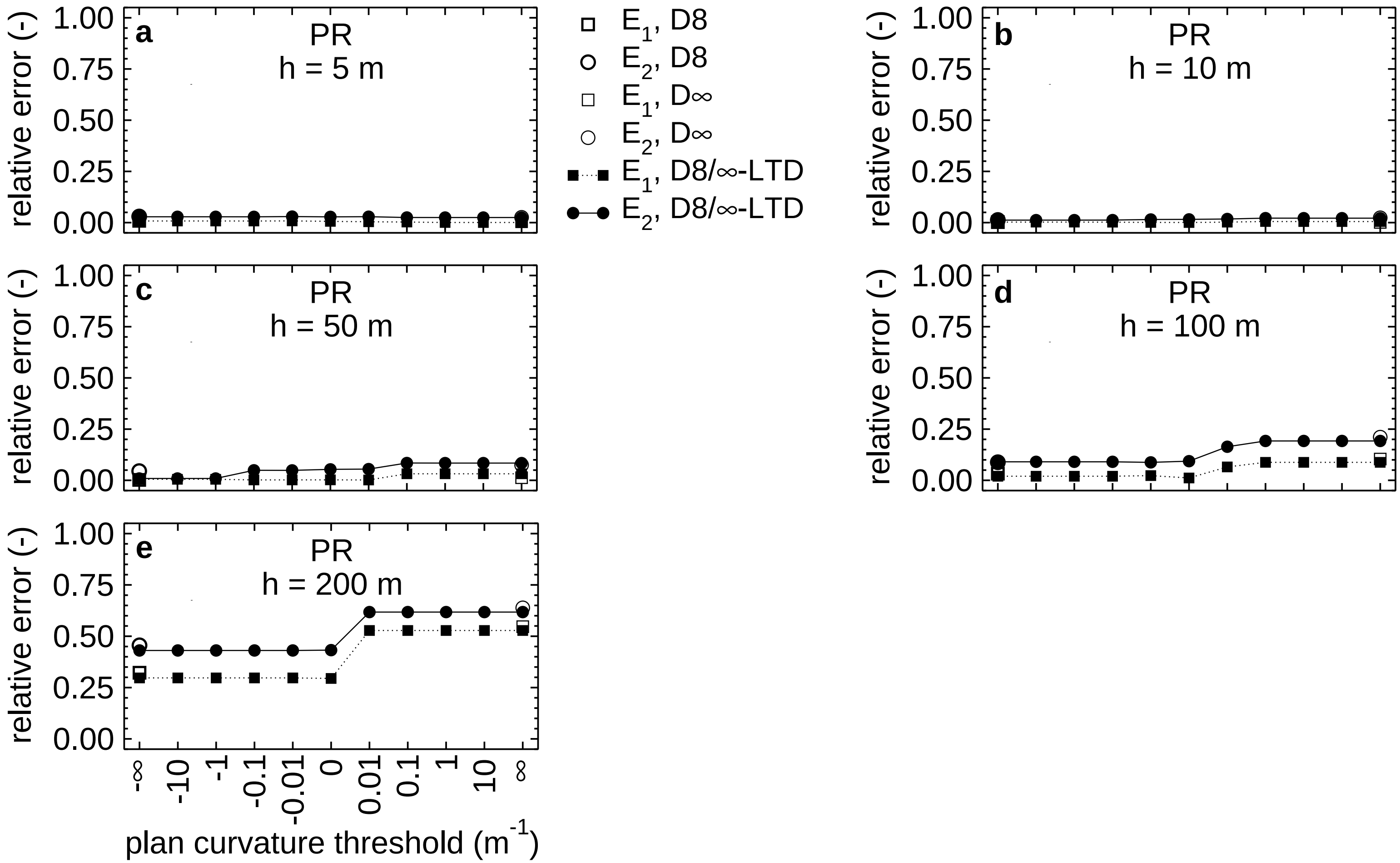
Drainage Basin and Subbasins



Slices Drained By One Grid Cell



Varying the Grid Cell Size



For drainage basins (and not for slopes) it is found that $E_2 \leq 10\%$ if $h \leq 0.15\, A^{0.4}$.

Conclusions

- Multiple flow direction (dispersive) methods, such as the D∞ and the D∞-LTD methods, are found in this study to produce smoother and more accurate between-cell variations of drainage area across divergent slopes compared to single flow direction (nondispersive) methods, such as the D8 and D8-LTD methods.
- These capabilities are however found to not necessarily reflect capabilities in the description of surface flow paths. Type 2 errors on drainage areas reveal that path-based nondispersive methods such as the D8-LTD method are equally or more accurate than dispersive methods in the description of surface flow paths along both convergent drainage basins.
- Both dispersive and nondispersive methods are found to be unable to provide an accurate description of surface flow paths along divergent slopes drained by line segments having length comparable to the grid cell size (type 2 errors exceeding 100%). Acceptable results are however obtained using path-based nondispersive methods when a divergent slope drained by more than one grid cell is globally considered (type 2 relative error equal to 12.3% for the DS).

The analysis carried out in this study reveals that path-based nondispersive methods are more effective than dispersive methods in describing surface flow paths, and therefore provide the soundest basis for a distributed description of gravity-driven processes.